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SPECIAL CORONAVIRUS
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PROPERTIES OF NORMAL DISTRIBUTIONS

(1) The 1st property is named REPRODUCTIVITY. It states that the sum of normal distributions also distributes according to the normal distribution, only if distributions that we add up are independent. For example, if daily sales distribute $N(\mu = 40, \sigma = 3)$, sales along will distribute also normal in this way:

$$N\left(\mu = 40 + 40 + 40 + 40 = 160,\ \sigma = \sqrt{3^2 + 3^2 + 3^2 + 3^2} = 6\right)$$

Remember that for σ we add up variances and not standard deviations!

(2) The 2nd property is about LINEAR TRANSFORMS. For example if production (P) and its cost (C) are related functionally in this way:

$$C = 10 + 2P$$

and production distributes following $N(\mu = 4, \sigma = 3)$, then cost distributes also normally in this way:

$$C \sim N(\mu = 10 + 2 \times 4,\ \sigma = 2 \times 3).$$

Remark that the adding constant doesn't influence the new σ.

Read the corresponding slides to get a more compact version of these properties.

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PROPERTIES OF NORMAL DISTRIBUTIONS
(problem solving).

3 types of situations may arise when solving problems about these properties:

(a) calculating a probability about the resulting distibution.

(b) calculating a maximum or minimum level for the resulting distribution with a given probability

(c) [only for reproductivity] calculating the number of normal distributions we must add up to reach a given level with a given probability [this is new, so pay attention]

For example, daily sales distribute $N(\mu = 40, \sigma = 3)$ and hence sales along 4 days distribute $N(\mu = 160, \sigma = 6)$ ~~and~~ as we know.

(a) what is the probability of selling more than 170?

$$P[X > 170] \underset{\text{std}}{=} P\left[z > \frac{170-160}{6}\right] = P[z > 1'66] =$$

$$= 1 - P[z < 1'66] \underset{\text{tables}}{=} 1 - 0'95154$$

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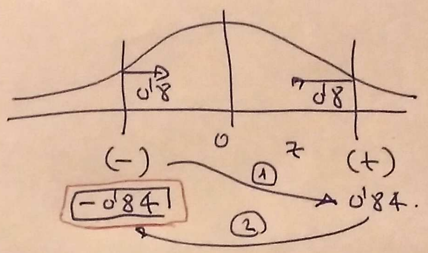
PROP. OF NORMAL DIST.
(problem distribution)

(b) Calculate a minimum value for sales with a 80% probability.

$$P[X > x] = P\left[Z > \frac{x-160}{6}\right] = 0'80$$

$\xrightarrow[\text{tables}]{}$ $\frac{x-160}{6} = -0'84 \longrightarrow \underline{x = 154'96}$

SOLUTION



(c) How many days do we need to get a sales level of 300 with a 0'99 probability?

X : daily sales $\sim N(\mu = 40, \sigma = 3) \xrightarrow[\text{along n days.}]{\text{reprod.}}$ $\cancel{X}$: total sales $\sim N(\mu = 40n, \sigma = 3\sqrt{n})$

$$P[\cancel{X} > 300] = 0'99$$
$$= P\left[Z > \frac{300-40n}{3\sqrt{n}}\right] = 0'99$$

$\downarrow$
std

Then : $\frac{300-40n}{3\sqrt{n}} = -2'32 \longrightarrow 40n - 0'96\sqrt{n} - 300 = 0$
using tables

$n = x^2$ $\quad 40x^2 - 0'96x - 300 = 0$

quadratic equation $\rightarrow$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \rightarrow$

$x = 2'82 \longrightarrow n = 7'95$ days $<$ 7 days $\rightarrow$ we are short! ; $\underline{\underline{8 \text{ days}}}$ $\rightarrow$ SOL

$x = -2'68 \longrightarrow x = \sqrt{n}$ and then would give a negative std dev.
[don't take]