Statistics for Business

Tests about one proportion

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PROBLEMS

159. In a market, 26% of the consumers buys our product. This year 44 consumers of 200 have declared they will follow to buy our product. With those data, draw a conclusion about the change in the proportion of consumers, by means of both the p-value and the critical region. Significance level: 5%.

First of all we have to set the null hypothesis. Ww have no explicit H_0, no question, no presumption. So let's look at data: data show that $p = \dfrac{44}{200} = 0.22$, lesser than usual $p = 0.26$, so it looks like the population proportion is lesser than 26%, so we take as the null hypothesis the opposite: $H_0 : p \geq 0.26$

We reject H_0 (big proportions) when sample proportions, namely $\hat{p}$, is *small*, so we have a perform a one-sided test on the lower side.

Under the null hypothesis, the sample proportion distributes in this way:

$$\hat{p} \sim N\left(0.26, \sqrt{\frac{0.26 \times 0.74}{200}} = 0.031\right)$$

Let's calculate the p-value:

$$p = P[\hat{p} < 0.22] = P\left[Z < \frac{0.22 - 0.26}{0.031}\right] = P[Z < -1.29] = 0.0985$$

As the p-value is bigger than the sigificance level, we accept the null hypothesis and consequently there is no reason to claim that the consumer proportion has changes.

Now let's solve by the critical method. Let's name the critical value for the sample proportion $\hat{p}_0$. As we reject the null hypothesis on the lower side:

$$P[\hat{p} < \hat{p}_0] = P\left[Z < \frac{\hat{p}_0 - 0.26}{0.031}\right] = 0.05 \rightarrow \frac{\hat{p}_0 - 0.26}{0.031} = -1.64 \rightarrow \hat{p}_0 = 0.209$$

So we will reject that the population proportion or real proportion is 0.26, when the $\hat{p}$ sample proportion is smaller than $\hat{p}_0 = 0.209$ (this is the critical value). As the sample proportion was finally 0.22, it's inside the acceptance region, so we have to accept the null hypothesis.
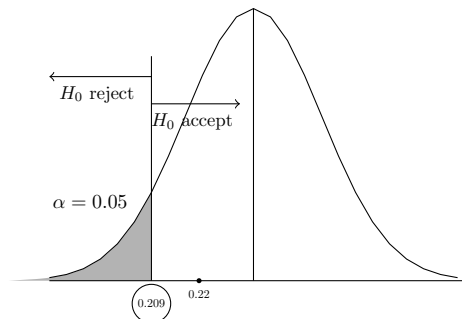
We can also interpret the test as a two-sided test, as in the problem statement we must drawn a conclusion about change, and change may happen on both sides.